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Composite polarization systems for independent controlling polarization of two beams with different wavelengths

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Abstract: The usage of independent and simultaneous control of the state of light polarization at different wavelengths can expand the capabilities of polarization methods for biomedical application. Unfortunately, all known methods of polarization conversion cannot convert the state of light polarization at different wavelengths independently. We propose a method and device for independent and simultaneous control of the polarization state at two wavelengths. We have theoretically proved the possibility of maintaining the phase shift at the first wavelength unchanged while simultaneously and independently changing the phase shift at the second wavelength from 0 to 180 degrees. The capabilities of the method were for the first time demonstrated for radiation with wavelengths $\lambda = 632.8$ nm and $\lambda = 488$ nm. At the wavelength $\lambda = 632.8$ nm, the phase shift remained equal to 180° whereas at the wavelength $\lambda = 488$ nm, it varied in the range from 121° to 136°.

1. Introduction

Polarized light methods, various types of polarimetry, and polarized light imaging in biomedical applications have sharply increased in recent years. They include developed laboratory techniques for biochemical and biomedical studies [1,2], histological analysis of tissue samples [3,4], ex vivo survey, and detection of diseases [5–9]. Polarization methods can be used for for determination of the optical parameters of collagen solutions [10], quantitative detection of biomarkers [11], for analysis of prostate adenoma and carcinoma diffuse tissues [12]. Mueller matrix polarimetry has been used for studies of spatial heterogeneities in hydrogel remodeling by smooth muscle cells [13]. Biosensors based on imaging ellipsometry have been used for antibody screening, hepatitis B markers detection, cancer markers spectrum determination, virus recognition, to image and quantify pathogenic viruses and disease biomarkers [14,15].

Independent and simultaneous control of the state of polarization at two wavelengths can expand the functionality of polarization methods for biomedical application, reduce the time for conducting research, and lead to the miniaturization of polarization devices operating at several wavelengths.

Unfortunately, all known methods of polarization conversion cannot convert the state of light polarization at different wavelengths independently. Optical systems that linearly transform polarization of light may be described in the formalism of 2×2 complex Jones matrices [16,17]. According to the generalized polarization matrix equivalence theorem [18], any polarization system comprising any number of polarization elements can be presented by only four optical elements. All those elements generally depend on the wavelength of light λ. The most prominent factor is the dependence of phase retardation Γ on the wavelength, namely, $\Gamma = \Delta nd/\lambda$, here Δn is the material birefringence, and d is the thickness of the retardation plate. That is why if an

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optical system transforms the polarization of light at one wavelength in a particular way, the change of polarization for another wavelength would be set and would depend on the values of the system parameters for that second wavelength. Thus, polarization states for two different wavelengths would become entangled.

The use of broadband (achromatic) polarization devices has been proposed to overcome this principal limitation. It turns out that combinations of half-wave and quarter-wave plates - if their main axes are oriented at particular angles - can equally transform the states of polarization in a wide range of wavelengths [19,20]. Increasing the number of quarter-wave and half-wave plates allows one to create broadband achromatic polarization systems with tunable phase retardation [21–24]. It is also possible to achieve broadband achromatic polarization devices using modern polymer and liquid crystal cells [25,26]. Combining phase plates with almost arbitrary phase retardations allows one to create polarization systems tunable in a wide wavelength range [27–31]. The system of LC spatial light modulators and a deformable mirror have been proposed to eliminate parasitic polarization inhomogeneities in the beam cross-section [32].

Thus, while much attention has been devoted to the development of various broadband polarization devices, the problem of independent and simultaneous conversion of the polarization of two-wave radiation has not been raised or properly studied. Yet, few separate issues on the subject have been experimentally addressed [33–35].

We propose a new method and device based on the use of several birefringent plates for simultaneous and independent polarization conversion of two beams with different wavelengths. The main principle of the method is based on the idea that several free parameters determine and govern the properties of composite polarization systems. It turns out that a set of four phase plates can make up a polarization system that simultaneously and independently transforms states of polarization for two wavelengths.

2. Theory of polarization systems

A polarization device based on a system of four phase plates is considered in our study (Fig. 1). Let the effective phase retardation of the system at the first wavelength λ_1 to be $\Gamma^{(1)}_{1,2,3,4} = \pi$. Let us find out if this is possible with the predetermined $\Gamma^{(2)}_{1,2,3,4}$ at the second wavelength λ_2. We will use the upper index (1) to denote all values related to the first wavelength and the upper index (2) to denote all values related to the second wavelength.

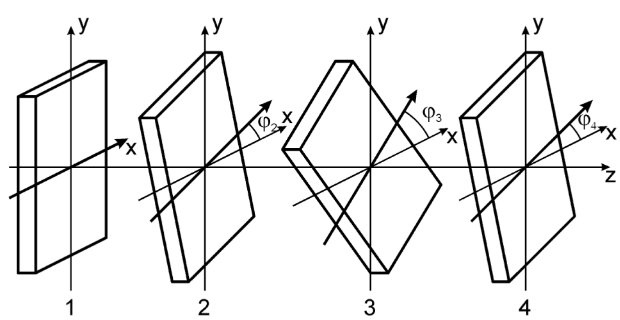

Fig. 1. Polarisation system consisted of four phase plates with the same retardation.

We chose the coordinate system so that the direction of light propagation coincides with the z-axis. The x-axis and the y-axis coincide with the direction of the slow axis and the fast axis of the first plate, respectively. The values φ_2, φ_3, and φ_4 denote the angles between the x-axis and the slow axes of the second, third, and fourth plates, respectively. The Jones matrices formalism [17] is used to analyze the polarization characteristics of the system. We will define the range of

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changes of system parameters, under which the condition $\Gamma^{(1)}_{1,2,3,4} = \pi$ is satisfied, and investigate the dependency of the value $\Gamma^{(2)}_{1,2,3,4}$ on the system parameters, when they are changed in this range.

We should use the modified Jones theorem [30] of the polarization systems equivalence to describe the properties of a four-component system. This theorem states that any polarization system consisting of two phase-retardation plates can be presented as a system of one effective phase-retardation plate, and a rotator of the plane of polarization (an element with optical activity). First, we consider the system [28] that consists of two phase-retardation plates whose slow axes are oriented at an angle φ. Such a two-component polarization system has the following Jones matrix:

$$\mathbf{W}(\Gamma_1, \Gamma_2, \varphi) = \mathbf{R}(-\varphi) \cdot \mathbf{T}(\Gamma_2) \cdot \mathbf{R}(\varphi) \cdot \mathbf{T}(\Gamma_1), \tag{1}$$

where

$$\mathbf{T}(\Gamma_i) = \begin{pmatrix} 1 & 0 \\ 0 & \exp(i\Gamma_i) \end{pmatrix}$$

is the polarization matrix of the phase retardation plate in the coordinate system associated with its eigen axes; Γ_1 and Γ_2 are phase retardations of the first and second plates, respectively. The rotation matrix $\mathbf{R}(\varphi)$ are used for transit from one coordinate system to another:

$$\mathbf{R}(\varphi) = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}. \tag{2}$$

The theorem of the equivalence [30] allows the matrix $\mathbf{W}(\Gamma_1, \Gamma_2, \varphi)$ to be equated to the matrix of the system consisting of one effective phase-retardation plate, and a rotator of the plane of polarization $\mathbf{V}(\varepsilon, \theta, \Gamma_{1,2})$:

$$\mathbf{W}(\Gamma_1, \Gamma_2, \varphi) = \mathbf{V}(\varepsilon, \theta, \Gamma_{1,2}). \tag{3}$$

Here $\mathbf{V}(\theta, \varepsilon, \Gamma_{1,2}) = \mathbf{R}(-\theta) \cdot \mathbf{R}(-\varepsilon) \cdot \mathbf{T}(\Gamma_{1,2}) \cdot \mathbf{R}(\varepsilon)$ where $\mathbf{R}(\pm\varepsilon)$, $\mathbf{R}(-\theta)$ are the rotation matrices defines like Eq. (2) , θ is an angle of the optical activity, $\Gamma_{1,2}$ is the effective phase retardation and ε is the angle between the eigen axis of the effective phase-retardation plate and the x-axis of the coordinate system. By solving Eq. (3), we can find the values of these variables

$$\begin{aligned} \Gamma_{1,2} &= \operatorname{acos}(\cos\Gamma_1 \cos\Gamma_2 - \cos 2\varphi \sin\Gamma_1 \sin\Gamma_2), \\ \varepsilon &= \frac{1}{2}\operatorname{atan}\left[\frac{\sin 2\varphi}{\cot\Gamma_2 \sin\Gamma_1 + \cos 2\varphi \cos\Gamma_1}\right], \\ \theta &= \operatorname{atan}\left[\frac{\sin 2\varphi}{\cot(\Gamma_1/2)\cot(\Gamma_2/2) - \cos 2\varphi}\right]. \end{aligned} \tag{4}$$

Further, we will consider the polarization properties of the four-component system for the first wavelength (Fig. 1). Let us assume for simplicity that the values of the phase retardations of all four plates are equal to each other and are equal to $\Gamma^{(1)}$ for the first wavelength and are equal to $\Gamma^{(2)}$ for the second wavelength. Thus, we can write the Jones matrix of the system as:

$$\begin{aligned} &\mathbf{W}^{(1)}(\Gamma^{(1)}, \varphi_2, \varphi_3, \varphi_4) = \\ &= \mathbf{R}(-\varphi_4) \cdot \mathbf{T}(\Gamma^{(1)}) \cdot \mathbf{R}(\varphi_4) \cdot \mathbf{R}(-\varphi_3) \cdot \mathbf{T}(\Gamma^{(1)}) \cdot \mathbf{R}(\varphi_3) \cdot \mathbf{R}(-\varphi_2) \cdot \mathbf{T}(\Gamma^{(1)}) \cdot \mathbf{R}(\varphi_2) \cdot \mathbf{T}(\Gamma^{(1)}), \end{aligned} \tag{5}$$

where $\mathbf{R}(\varphi_i)$ is the rotation matrix ($i = 2, 3, 4$), and $\mathbf{T}(\Gamma^{(1)})$ is the matrix of the single phase-retardation plate.

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Consistently applying the above equivalence theorem to the Eq. (5), it is possible to bring the polarization matrix of the system to a more convenient form for analysis. Taking into account the property of the rotation matrices $\mathbf{R}(\psi_1)\cdot\mathbf{R}(\psi_2)=\mathbf{R}(\psi_1+\psi_2)$, we can rewrite Eq. (5) as

$$\begin{aligned}
&\mathbf{W}^{(1)}(\Gamma^{(1)},\varphi_2,\varphi_3,\varphi_4)=\\
&=\mathbf{R}(-\varphi_4)\cdot\mathbf{T}(\Gamma^{(1)})\cdot\mathbf{R}(\varphi_4-\varphi_3)\cdot\mathbf{T}(\Gamma^{(1)})\cdot\mathbf{R}(\varphi_3)\cdot\mathbf{R}(\theta_{1,2}^{(1)}-\varepsilon_{1,2}^{(1)})\cdot\mathbf{T}(\Gamma_{1,2}^{(1)})\cdot\mathbf{R}(\varepsilon_{1,2}^{(1)})=\\
&=\mathbf{R}(-\varphi_4)\cdot\mathbf{T}(\Gamma^{(1)})\cdot\mathbf{R}(\varphi_4+\theta_{1,2}^{(1)}-\varepsilon_{1,2}^{(1)})\cdot\mathbf{R}(-\gamma_{1,2}^{(1)})\cdot\mathbf{T}(\Gamma^{(1)})\cdot\mathbf{R}(\gamma_{1,2}^{(1)})\cdot\mathbf{T}(\Gamma_{1,2}^{(1)})\cdot\mathbf{R}(\varepsilon_{1,2}^{(1)})=\\
&=\mathbf{R}(-\varphi_4)\cdot\mathbf{T}(\Gamma^{(1)})\cdot\mathbf{R}(\varphi_4+\theta_{1,2}^{(1)}-\varepsilon_{1,2}^{(1)})\cdot\mathbf{R}(\theta_{1,2,3}^{(1)}-\varepsilon_{1,2,3}^{(1)})\cdot\mathbf{T}(\Gamma_{1,2,3}^{(1)})\cdot\mathbf{R}(\varepsilon_{1,2,3}^{(1)})\cdot\mathbf{R}(\varepsilon_{1,2}^{(1)})=\\
&=\mathbf{R}(-\varphi_4)\cdot\mathbf{T}(\Gamma^{(1)})\cdot\mathbf{R}(\beta^{(1)})\cdot\mathbf{T}(\Gamma_{1,2,3}^{(1)})\cdot\mathbf{R}(\varepsilon_{1,2,3}^{(1)}+\varepsilon_{1,2}^{(1)}).
\end{aligned}\tag{6}$$

Here, the ε with indices denote the angles between the slow axes of the effective retardation plates and the x-axis. The variables θ with indices indicate the azimuths of the plane of polarization rotation by the effective rotator. $\Gamma^{(1)}$ with indices denote the effective phase retardations of the corresponding polarization systems. The subscripts 1,2 and 1,2,3 indicate the parameters related to the polarization systems consisting of the first and second or the first, second and third plates. The variable $\gamma^{(1)}=\varphi_3+\theta_{1,2}^{(1)}-\varepsilon_{1,2}^{(1)}$ denotes the angle between the effective slow axis of the composite polarization system 1,2 and the slow axis of the third retardation plate, thus taking into account the presence of the effective optical activity $\theta_{1,2}$ for the system 1,2; while $\beta^{(1)}=\varphi_4+\theta_{1,2}^{(1)}+\theta_{1,2,3}^{(1)}-\varepsilon_{1,2}^{(1)}-\varepsilon_{1,2,3}^{(1)}$ has the same definition only for the system 1,2,3 and the fourth plate.

The variables in Eq. (6) are related to each other in the same way as in Eq. (4):

$$\Gamma_{1,2}^{(1)}=\operatorname{acos}\left(\cos^2\Gamma^{(1)}-\cos 2\varphi_2\sin^2\Gamma^{(1)}\right),\tag{7}$$

$$\varepsilon_{1,2}^{(1)}=\frac{1}{2}\operatorname{atan}\left[\frac{\sin 2\varphi_2}{(1+\cos 2\varphi_2)\cos\Gamma^{(1)}}\right],\tag{8}$$

$$\theta_{1,2}^{(1)}=\operatorname{atan}\left[\frac{\sin 2\varphi_2}{\cot^2(\Gamma^{(1)}/2)-\cos 2\varphi_2}\right],\tag{9}$$

$$\Gamma_{1,2,3}^{(1)}=\operatorname{acos}\left(\cos\Gamma^{(1)}\cos\Gamma_{1,2}^{(1)}-\cos 2\gamma^{(1)}\sin\Gamma^{(1)}\sin\Gamma_{1,2}^{(1)}\right),\tag{10}$$

$$\varepsilon_{1,2,3}^{(1)}=\frac{1}{2}\operatorname{atan}\left(\frac{\sin 2\gamma^{(1)}}{\cot\Gamma^{(1)}\sin\Gamma_{1,2}^{(1)}+\cos 2\gamma^{(1)}\cos\Gamma_{1,2}^{(1)}}\right),\tag{11}$$

$$\theta_{1,2,3}^{(1)}=\operatorname{atan}\left[\frac{\sin 2\gamma^{(1)}}{\cot(\Gamma_{1,2}^{(1)}/2)\cot(\Gamma^{(1)}/2)-\cos 2\gamma^{(1)}}\right].\tag{12}$$

Further, it is necessary to find the values of the system parameters when the system has phase retardation $\Gamma_{1,2,3,4}^{(1)}=\pi$ at the first wavelength. For this purpose, we note that according to the last line of Eq. (6), the polarization system under investigation is equivalent to the consecutively placed effective phase-retardation plate with $\Gamma_{1,2,3}^{(1)}$ and the fourth phase-retardation plate with $\Gamma^{(1)}$. Such polarization system can have the effective phase retardation π only if the angle $\beta^{(1)}$ between the slow axes of these plates is zero, and the sum of their phase retardations is equal to π. It means that the condition $\Gamma_{1,2,3,4}^{(1)}=\pi$ is satisfied for those values of the system parameters

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for which the following equalities are valid:

$$\varphi_4 = \varepsilon_{1,2}^{(1)} + \varepsilon_{1,2,3}^{(1)} - \theta_{1,2}^{(1)} - \theta_{1,2,3}^{(1)}, \tag{13}$$

$$\Gamma_{1,2,3}^{(1)} + \Gamma^{(1)} = \pi. \tag{14}$$

From Eq. (10) and Eq. (14) we obtain the following equation:

$$-\cos\Gamma^{(1)} = \cos\Gamma^{(1)}\cos\Gamma_{1,2}^{(1)} - \cos 2\gamma^{(1)}\sin\Gamma^{(1)}\sin\Gamma_{1,2}^{(1)}. \tag{15}$$

From Eq. (15), it is possible to obtain an expression for the value of the angle $\gamma^{(1)}$:

$$\gamma^{(1)} = \frac{1}{2}\mathrm{acos}\left(\cot\Gamma^{(1)}\frac{1+\cos\Gamma_{1,2}^{(1)}}{\sin\Gamma_{1,2}^{(1)}}\right). \tag{16}$$

Since according to Eqs. (7) the values $\Gamma_{1,2}^{(1)}$ depend only on the angle φ_2 for the fixed $\Gamma^{(1)}$ then $\gamma^{(1)}$ is a function of φ_2. The angle φ_3 can also be expressed in terms of $\gamma^{(1)}$, $\varepsilon_{1,2}^{(1)}$, $\theta_{1,2}^{(1)}$ and it is also a function of φ_2:

$$\varphi_3 = \gamma^{(1)} + \varepsilon_{1,2}^{(1)} - \theta_{1,2}^{(1)}. \tag{17}$$

The angle φ_4 is expressed in Eq. (13), and it is also a function of φ_2. Thus, if the four-component system has phase shift $\Gamma_{1,2,3,4}^{(1)} = \pi$, then all the other system parameters (φ_3 and φ_4) are uniquely determined for the given parameter φ_2. It means that there is only one independent free parameter (the angle φ_2) for the system.

Now we investigate how the effective phase retardation of the four-component system at the second wavelength changes when the parameters of the system are varied. Since the polarization properties of the system under investigation are uniquely determined by the angles φ_2, φ_3, φ_4 and phase retardation of its components for the chosen wavelength, so the effective phase retardation $\Gamma_{1,2,3,4}^{(2)}$ of the system for the second wavelength is a function of the angle φ_2 and quantities $\Gamma^{(1)}$ and $\Gamma^{(2)}$, when the system is adjusted according to the condition $\Gamma_{1,2,3,4}^{(1)} = \pi$.

Equation (5) remains valid for the second wavelength when replacing $\Gamma^{(1)}$ with $\Gamma^{(2)}$. Then the values of the angles φ_3 and φ_4 are determined from Eq. (13), Eq. (17) as a function of φ_2 and $\Gamma^{(1)}$. Then, the modified equivalence theorem Eq. (3) is applied to the resulting polarization matrix, taking into account the indicated substitutions. Having done these calculations, we get the following equation from Eq. (6):

$$\begin{aligned}
&\mathbf{W}^{(2)}(\varphi_2, \Gamma^{(1)}, \Gamma^{(2)}) = \\
&= \mathbf{R}(\theta_{1,2}^{(2)} + \theta_{1,2,3}^{(2)} - \varepsilon_{1,2}^{(2)} - \varepsilon_{1,2,3}^{(2)}) \cdot \mathbf{R}(-\beta^{(2)}) \cdot \mathbf{T}(\Gamma^{(2)}) \cdot \mathbf{R}(\beta^{(2)}) \cdot \mathbf{T}(\Gamma_{1,2,3}^{(2)}) \cdot \mathbf{R}(\varepsilon_{1,2}^{(2)} + \varepsilon_{1,2,3}^{(2)}) = \\
&= \mathbf{R}(\theta_{1,2}^{(2)} + \theta_{1,2,3}^{(2)} + \theta_{1,2,3,4}^{(2)} - \varepsilon_{1,2}^{(2)} - \varepsilon_{1,2,3}^{(2)} - \varepsilon_{1,2,3,4}^{(2)}) \cdot \mathbf{T}(\Gamma_{1,2,3,4}^{(2)}) \cdot \mathbf{R}(\varepsilon_{1,2}^{(2)} + \varepsilon_{1,2,3}^{(2)} + \varepsilon_{1,2,3,4}^{(2)}).
\end{aligned} \tag{18}$$

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The expressions for variables with the superscript (2) in Eq. (18) are essentially the same as Eqs. (7) –(12) for the first wavelength only substituting superscripts (1) for (2). Here

$$
\begin{aligned}
\Gamma_{1,2}^{(2)} &= \operatorname{acos}\left(\cos^2\Gamma^{(2)} - \cos 2\varphi_2 \sin^2\Gamma^{(2)}\right), \\
\varepsilon_{1,2}^{(2)} &= \tfrac{1}{2}\operatorname{atan}\left[\frac{\sin 2\varphi_2}{(1+\cos 2\varphi_2)\cos\Gamma^{(2)}}\right], \\
\theta_{1,2}^{(2)} &= \operatorname{atan}\left[\frac{\sin 2\varphi_2}{\cot^2(\Gamma^{(2)}/2)-\cos 2\varphi_2}\right], \\
\gamma^{(2)} &= \varphi_3 - \varepsilon_{1,2}^{(2)} + \theta_{1,2}^{(2)}, \\
\Gamma_{1,2,3}^{(2)} &= \operatorname{acos}\left(\cos\Gamma^{(2)}\cos\Gamma_{1,2}^{(2)} - \cos 2\gamma^{(2)}\sin\Gamma^{(2)}\sin\Gamma_{1,2}^{(2)}\right), \\
\varepsilon_{1,2,3}^{(2)} &= \tfrac{1}{2}\operatorname{atan}\left(\frac{\sin 2\gamma^{(2)}}{\cot\Gamma^{(2)}\sin\Gamma_{1,2}^{(2)}+\cos 2\gamma^{(2)}\cos\Gamma_{1,2}^{(2)}}\right), \\
\theta_{1,2,3}^{(2)} &= \operatorname{atan}\left[\frac{\sin 2\gamma^{(2)}}{\cot(\Gamma_{1,2}^{(2)}/2)\cot(\Gamma^{(2)}/2)-\cos 2\gamma^{(2)}}\right]. \\
\beta^{(2)} &= \varphi_4 - \varepsilon_{1,2}^{(2)} - \varepsilon_{1,2,3}^{(2)} + \theta_{1,2}^{(2)} + \theta_{1,2,3}^{(2)}, \\
\Gamma_{1,2,3,4}^{(2)} &= \operatorname{acos}\left(\cos\Gamma_{1,2,3}^{(2)}\cos\Gamma^{(2)} - \cos 2\beta^{(2)}\sin\Gamma_{1,2,3}^{(2)}\sin\Gamma^{(2)}\right), \\
\varepsilon_{1,2,3,4}^{(2)} &= \tfrac{1}{2}\operatorname{atan}\left(\frac{\sin 2\beta^{(2)}}{\cot\Gamma^{(2)}\sin\Gamma_{1,2,3}^{(2)}+\cos 2\beta^{(2)}\cos\Gamma_{1,2,3}^{(2)}}\right), \\
\theta_{1,2,3,4}^{(2)} &= \operatorname{atan}\left[\frac{\sin 2\beta^{(2)}}{\cot(\Gamma_{1,2,3}^{(2)}/2)\cot(\Gamma^{(2)}/2)-\cos 2\beta^{(2)}}\right].
\end{aligned}
\tag{19}
$$

Thus, we have obtained all the relationships necessary to determine the conditions under which an adjustable polarization system consisting of four phase-retardation plates has the effective phase retardation π for the first wavelength and any requiring phase retardation $\Gamma_{1,2,3,4}^{(2)}$ for the second wavelength by varying independent free parameter (the angle φ_2).

Figure 2 shows the dependence of the effective phase retardation $\Gamma_{1,2,3,4}^{(2)}$ on the adjusting angle φ_2 for values $\Gamma^{(1)} = 55°$, $\Gamma^{(2)} = 125°$, provided that the system has $\Gamma_{1,2,3,4}^{(1)} = \pi$. It can be seen from Fig. 2 that when the adjusting angle φ_2 is changed, the effective phase retardation $\Gamma_{1,2,3,4}^{(2)}$ continuously varies in the range from almost 0 to 180°.

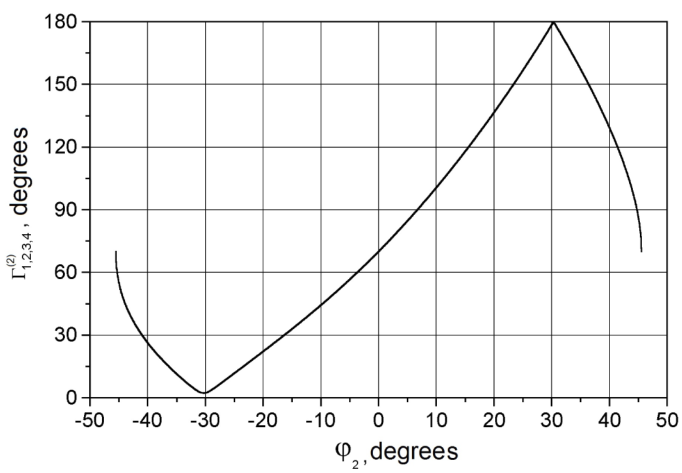

Fig. 2. The dependence of the effective phase retardation $\Gamma_{1,2,3,4}^{(2)}$ on the adjusting angle φ_2 at the values $\Gamma^{(1)} = 55°$, $\Gamma^{(2)} = 125°$.

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3. Experimental results

The described approach was used to study experimentally a system consisting of four phase retardation plates with equal phase retardation. The phase retardation of the birefringent plates was chosen to be $\Gamma^{(1)} = 52.8°$ at the first wavelength $\lambda_1 = 632.8$ nm and $\Gamma^{(2)} = 72.9°$ at the second wavelength $\lambda_2 = 488$ nm. The tuning of the system to have a certain effective phase retardation at the second wavelength was performed by changing the angle φ_2 (the free parameter). The phase retardation of the system at the second wavelength was measured and calculated, assuming the system has phase shift π at the first wavelength.

The experimental setup is shown in Fig. 3. A helium-neon laser (the wavelength $\lambda_1 = 632.8$ nm) and an argon ion laser ($\lambda_1 = 488$ nm) were used as light sources. Both lasers were generated in the fundamental transverse mode. The radiation of the helium-neon laser had a sufficiently high degree of linear polarization. The intensity-related ellipticity of its polarization e^I was 10^{-4}. The argon laser radiation was almost completely depolarized. The polarizer P1 (Glan prism) was used to produce linearly polarized light at the wavelength $\lambda_2 = 488$ nm. A quarter wave plate converted the linearly polarized radiation at both wavelengths into circularly polarized. By turning polarizer P_2 (Glan prism), it was possible to set the required azimuth of the plane of polarization of linearly polarized light at the input of the four-element polarization system under investigation. An analyzer A was installed after the polarization system. The polarization system consisted of four identical mica waveplates. The retardations of the waveplates at both wavelengths ($\Gamma^{(1)} = 52.8°$ and $\Gamma^{(2)} = 72.9°$) were measured using our method described in [36]. Using the values of the parameters $\Gamma^{(1)}$ and $\Gamma^{(2)}$, we calculated the values of the angles φ_2, φ_3 and φ_4 and installed the retardation plates.

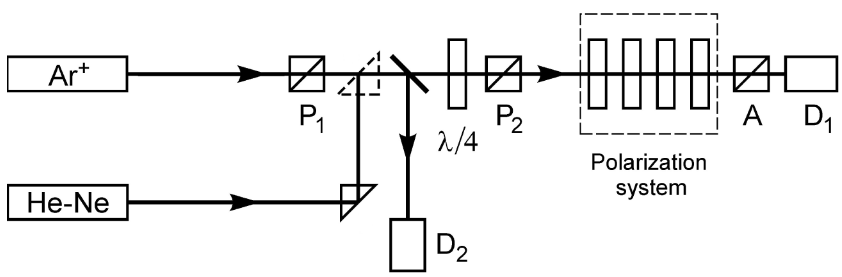

Fig. 3. Experimental setup.

At first, we proved that the polarization system has the property of a half-wave plate at the first wavelength $\lambda_1 = 632.8$ nm for the different sets of the angles φ_2, φ_3 and φ_4. We determined the azimuth of the effective eigen axes $\varepsilon^{(1)}_{1,2,3,4}$ and the angle of the effective optical activity of such system $\theta^{(1)}_{1,2,3,4}$. The quality and azimuth of the linearly polarized light at the output of the system were checked for the different orientations of the input linear polarization. We had high-quality linear polarization at the system output; the intensity-related ellipticity of polarization e^I did not exceed $\sim 10^{-4}$. The azimuth of the linearly polarized light changed in the same way as after an ordinary half-wave plate if we take into account additional optical activity $\theta^{(1)}_{1,2,3,4}$.

The dependence of the azimuth of one of the eigen axes $\varepsilon^{(1)}_{1,2,3,4}$ on the adjusting angle φ_2 is shown in Fig. 4. Fig. 4 shows that the value $\varepsilon^{(1)}_{1,2,3,4}$ depends on the adjusting angle φ_2 and experimentally measured values of $\varepsilon^{(1)}_{1,2,3,4}$ are in good agreement with the calculated values.

Three polarization parameters of the polarization system, namely, the direction of one of the effective eigen axes $\varepsilon^{(2)}_{1,2,3,4}$, effective optical activity $\theta^{(2)}_{1,2,3,4}$ and effective phase retardation $\Gamma^{(2)}_{1,2,3,4}$ at wavelength $\lambda_2 = 488$ nm were measured for the different sets of angles φ_2, φ_3 and

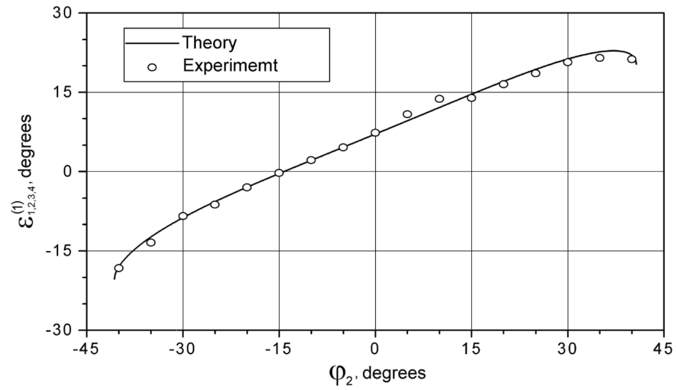

Fig. 4. The dependence of the azimuth $\varepsilon^{(1)}_{1,2,3,4}$ of one of the polarization system axes on the adjusting angle φ_2 at the wavelength λ_1.

φ_4. The effective phase retardation $\Gamma^{(2)}_{1,2,3,4}$ of the system under investigation was determined by measuring the maximum ellipticity, which the system provides. To determine the ellipticity e^I we rotated the analyzer and measured the maximum and minimum light intensity passed through the polarization system. The value of e^I was estimated as the ratio of the minimum and maximum measured intensity. To exclude the influence of the lasers output power fluctuations, the output intensity was determined as the ratio of signals from photodiodes D_1 and D_2.

Figure 5 shows the dependence of the polarization system effective phase retardation $\Gamma^{(2)}_{1,2,3,4}$ on the adjusting angle φ_2. The obtained experimental results are in good agreement with theoretical data. The observed deviation of experimental results from the calculated ones could be due to neglecting the linear amplitude anisotropy in the theoretical calculations.

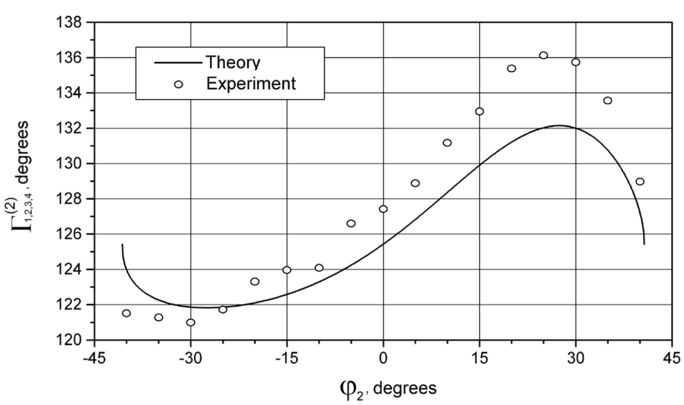

Fig. 5. The dependence of the effective phase retardation $\Gamma^{(2)}_{1,2,3,4}$ of the polarization system on the adjusting angle φ_2 at wavelength λ_2.

Thus, the experimental results demonstrate the validity of a new method for independent and simultaneous polarization control of two light beams with different wavelengths. It should be stressed that achieving the effective phase retardation at the second wavelength up to $\Gamma^{(2)}_{1,2,3,4} = 180°$ is possible using LC-cell as a retardation plate as was theoretically shown in our paper [37].

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4. Conclusions

We have proposed a composite polarization system for independent and simultaneous polarization control of two light beams with different wavelengths. It was shown that the polarization system should consist of four phase plates with the same birefringence but with different axes orientations relative to each other. We were successful in demonstrating the independent and simultaneous polarization control at wavelength $\lambda_1 = 632.8$ nm and $\lambda_2 = 488$ nm keeping the retardation at the first wavelength $\Gamma^{(1)}_{1,2,3,4} = 180°$ unchanged while the retardation at the second wavelength changed in the range from $\Gamma^{(2)}_{1,2,3,4} = 121°$ to $\Gamma^{(2)}_{1,2,3,4} = 136°$. To extend this range, it is necessary to use the elements with variable phase shift (LC-cell) as the components of the system.

In many biomedical applications of optical scanning and tomography, the use of multiple wavelengths is required to increase sensitivity and accuracy of measurements. On the other hand, the great advantage of proposed approach here is clear possibility for miniaturization of desired polarization systems. For example, the use of dual-wavelength optical polarimetry is considered extensively for real-time, non-invasive in-vivo glucose sensing [38–40]. Implementation of our approach for designing polarization systems may help to develop these table-top experimental prototypes into real palm-sized devices for clinical use.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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